4 . Geometry

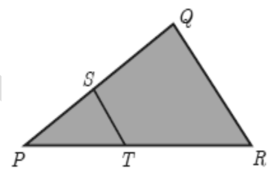
1 mark Questions

1. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when
   (A) $\angle B = \angle E$ (B) $\angle A = \angle D$ **(C) $\angle B = \angle D$** (D) $\angle A = \angle F$

2. In ΔLMN, $\angle L = 60^\circ$, $\angle M = 50^\circ$. If $\Delta LMN \sim \Delta PQR$ then the value of $\angle R$ is SEP-20
   (A) 40° **(B) 70°** (C) 30° (D) 110°

3. If ΔABC is an isosceles triangle with $\angle C = 90^\circ$ and $AC = 5\ cm$, then AB is PTA-4, MAY-22
   (A) $2.5\ cm$ (B) $5\ cm$ (C) $10\ cm$ **(D) $5\sqrt{2}\ cm$**

4. In a given figure $ST \parallel QR$, $PS = 2\ cm$ and $SQ = 3\ cm$. Then the ratio of the area of ΔPQR to the area of ΔPST is
   (A) 25 : 4 (B) 25 : 7
   (C) 25 : 11 (D) 25 : 13

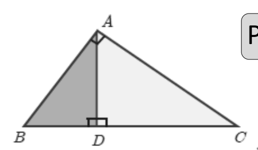


5. The perimeters of two similar triangles ΔABC and ΔPQR are $36\ cm$ and $24\ cm$ respectively. If $PQ = 10\ cm$, then the length of AB is PTA-5
   (A) $6\frac{2}{3}\ cm$ (B) $\frac{10\sqrt{6}}{3}\ cm$ (C) $66\frac{2}{3}\ cm$ **(D) $15\ cm$**

6. If in ΔABC, $DE \parallel BC$. $AB = 3.6\ cm$, $AC = 2.4\ cm$ and $AD = 2.1\ cm$ then the length of AE is
   (A) $1.4\ cm$ (B) $1.8\ cm$ (C) $1.2\ cm$ (D) $1.05\ cm$ SEP-21, PTA-3, JUL-22

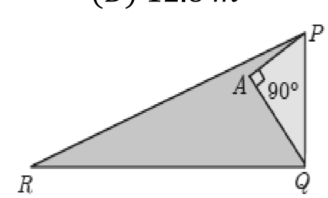
7. In a ΔABC, AD is the bisector of $\angle BAC$. If $AB = 8\ cm$, $BD = 6\ cm$ and $DC = 3\ cm$. The length of the side AC is PTA-6, MAY-22
   (A) $6\ cm$ **(B) $4\ cm$** (C) $3\ cm$ (D) $8\ cm$

8. In the adjacent figure $\angle BAC = 90^\circ$ and $AD \perp BC$ then PTA-1
   (A) $BD \cdot CD = BC^2$ (B) $AB \cdot AC = BC^2$
   (C) $BD \cdot CD = AD^2$ (D) $AB \cdot AC = AD^2$



9. Two poles of heights $6\ m$ and $11\ m$ stand vertically on a plane ground. If the distance between their feet is $12\ m$, what is the distance between their tops?
   (A) $13\ m$ (B) $14\ m$ (C) $15\ m$ (D) $12.8\ m$

10. In the given figure, $PR = 26\ cm$, $QR = 24\ cm$, $\angle PAQ = 90^\circ$, $PA = 6\ cm$ and $QA = 8\ cm$. Find $\angle PQR$ PTA-6
    (A) 80° (B) 85°
    (C) 75° **(D) 90°**



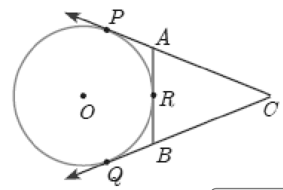
11. A tangent is perpendicular to the radius at the PTA-2
    (A) centre **(B) point of contact** (C) infinity (D) chord

12. How many tangents can be drawn to the circle from an exterior point? SEP-21, JUL-22
    (A) one **(B) two** (C) infinite (D) zero

13. The two tangents from an external points P to a circle with centre at O are PA and PB. If $\angle APB = 70^\circ$ then the value of $\angle AOB$ is

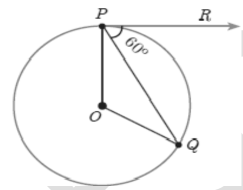
(A) 100° **(B) 110°** (C) 120° (D) 130°

14. If figure CP amd CQ are tangents to a circle with centre at O. ARB is another tangent touching the circle at R. If $CP = 11\ cm$ and $BC = 7\ cm$, then the length of BR is MDL



(A) $6\ cm$ (B) $5\ cm$ (C) $8\ cm$ **(D) $4\ cm$**

15. In figure if PR is tangent to the circle at P and O is the centre of the circle, then $\angle POQ$ is SEP-20



(A) 120° (B) 100°

(C) 110° (D) 90°

2 mark Questions

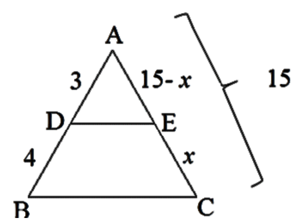
1. **In ΔABC, D and E are points on the sides AB and AC respectively such that $DE||BC$**

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15cm$ find AE. SEP-21

Given in $\Delta ABC, D$ and E are points an the sides AB & AC respectively such that $DE||BC$

$\therefore$ By Thales theorem, $\frac{AD}{DB} = \frac{AE}{EC}$

Let $EC = x$, $AE = 15 - x$



$\frac{3}{4} = \frac{15-x}{x}$

$3x = 60 - 4x$

$3x + 4x = 60$

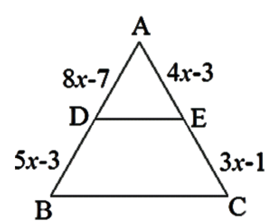
$7x = 60$

$x = \frac{60}{7} = 8.57$

$AE = 15 - 8.57 = \mathbf{6.43cm}$

(ii) If $AD = 8x - 7$, $DB = 5x - 3$, $AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

By Thales theorem, $\frac{AD}{DB} = \frac{AE}{EC}$



$\frac{8x-7}{5x-3} = \frac{4x-3}{3x-1}$

$(8x-7)(3x-1) = (4x-3)(5x-3)$

$24x^2 - 8x - 21x + 7 = 20x^2 - 12x - 15x + 9$

$4x^2 - 2x - 2 = 0$

$\div 2 \Rightarrow \quad 2x^2 - x - 1 = 0$

$(x-1)(2x+1) = 0$

$x = 1$ (or) $x = -\frac{1}{2} \Rightarrow \boldsymbol{x = 1}$

Since $x \neq -\frac{1}{2}$

2. ABCD is a trapezium which $AB||DC$ and P, Q are points on AD and BC respectively, such that $PQ||DC$ if $PD = 18cm$, $BQ = 35cm$ and $QC = 15cm$, and find AD. JUL-22

In trapezium $ABCD$, $AB \parallel CD \parallel PQ$

Join AC, meets PQ at R

In ΔACD, $PR \parallel CD$

By BPT, $\frac{AP}{PD} = \frac{AR}{RC}$

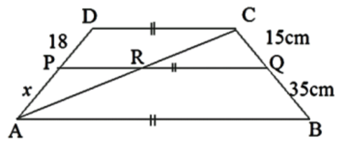
$\frac{x}{18} = \frac{AR}{RC}$ ....(1)

In ΔABC, $RQ \parallel AB$

By BPT, $\frac{BQ}{QC} = \frac{AR}{RC}$

$\frac{35}{15} = \frac{AR}{RC}$

$\frac{7}{3} = \frac{AR}{RC}$ ....(2)



From (1) and (2), $\frac{x}{18} = \frac{7}{3}$

$3x = 126$

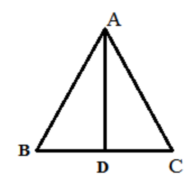
$x = \frac{126}{3} = 42$

If $AP = x$

$AP = 42$

$AD = AP + PD = 42 + 18 =$ **60 cm**

3. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following SEP-20



(i) $AB = 5cm, AC = 10cm, BD = 1.5cm$ and $CD = 3.5cm$

Given: In the ΔABC,

$\frac{AB}{AC} = \frac{5}{10}$

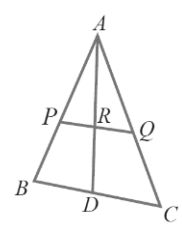
$\frac{AB}{AC} = \frac{1}{2}$.................... (1)

$\frac{BD}{DC} = \frac{1.5}{3.5}$

$\frac{BD}{DC} = \frac{15}{35}$

$\frac{BD}{DC} = \frac{3}{7}$ .... (2)

(1) & (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{DC}$



$\therefore$ AD is **not an angle bisector** of $\angle A$

(ii) $AB = 4cm$, $AC = 6cm, BD = 1.6cm$ and $CD = 2.4cm$

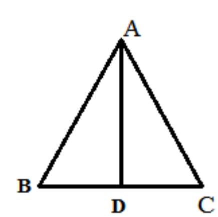
$\frac{AB}{AC} = \frac{4}{6}$

$\frac{AB}{AC} = \frac{2}{3}$............... (1)

$\frac{BD}{CD} = \frac{1.6}{2.4}$

$\frac{BD}{CD} = \frac{16}{24}$

$\frac{BD}{CD} = \frac{2}{3}$.............. (2)



(1)& (2) $\frac{AB}{AC} = \frac{BD}{CD}$

$\Rightarrow$ AD is the angle **bisector** of $\angle A$

4. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point? JUL-22

Given, $BC = 18\text{m}, BA = 24\text{m}$

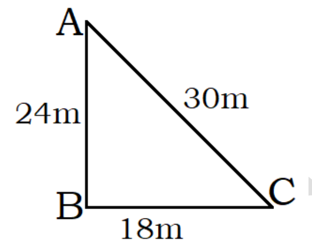
By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$= 24^2 + 18^2$

$= 576 + 324$

$AC^2 = 900 = 30^2$

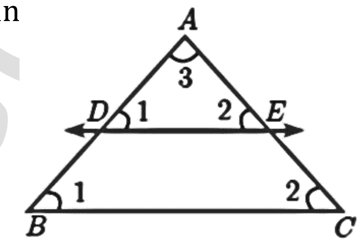
$\boldsymbol{AC = 30m}$



5 mark Questions

Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem MAY-22

Statement: A straight line drawn parallel to a side of triangle intersectin the other two sides, divides the sides in the same ratio.



Proof:

Given: In ΔABC, D is a point on AB and E is a point on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \sim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	By AAA similarity Corresponding sides are proportional Split AB and AC using the points D and E On simplification Cancelling 1 on both sides Taking reciprocals
Hence proved		

Corollary: If in ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then

(i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

2. Theorem 3: Angle Bisector Theorem PTA-5,SEP-20, JUL-22

Statement: The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Proof:

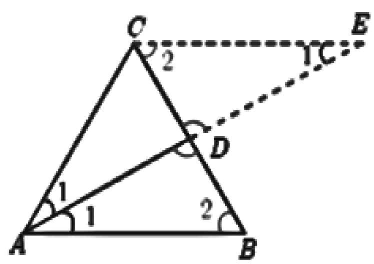
Given : In ΔABC, AD is the internal bisector

To prove:

$$\frac{AB}{AC} = \frac{BD}{CD}$$

Construction :

Draw a line through C parallel to AB. Extend AD to meet line through C at E

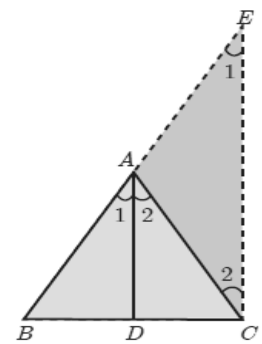


No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles $AC = CE$ …… (1)	In ΔACE, $\angle CAE = \Delta CEA$
3.	$\Delta ABD \backsim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved.

3. Theorem 4: Converse of Angle Bisector Theorem

PTA-3, 4

Statement: If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.



Proof:

Given : ABC is a triangle.

AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

That is $\frac{AB}{AC} = \frac{BD}{DC}$ ……… (1)

To prove : AD bisects $\angle A$ i.e. $\angle 1 = \angle 2$

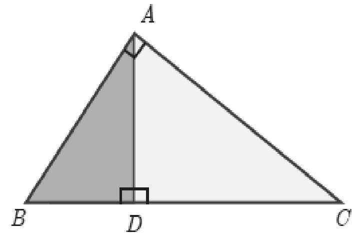
Construction : Draw $CE \parallel DA$. Extend BA to meet at E.

No.	Statement	Reason
1.	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC = \angle 1$	Since $DA \parallel CE$ and AC is transversal, corresponding angles are equal
3.	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$ and AC is transversal, Alternate angles are equal
4.	$\frac{BA}{AE} = \frac{BD}{DC}$ ……….. (2)	In ΔBCE by thales theorem
5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7.	$AC = AE$……….(3)	Cancelling AB
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9.	AD bisects $\angle A$	Since, $\angle 1 = \angle BAD = \angle 2 = \angle DAC$. Hence proved

4. Theorem 5: Pythagoras Theorem

SEP-21, PTA-4

Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Proof:

Given : In $\Delta ABC,\ \angle A = 90°$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \backsim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ ...(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity
2.	Compare ΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \backsim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle ADC = 90°$ By AA similarity

Adding (1) and (2) we get

$$AB^2 + AC^2 = (BC \times BD) + (BC \times DC)$$
$$= BC \times (BD + DC)$$
$$= BC \times BC$$
$$AB^2 + AC^2 = BC^2$$

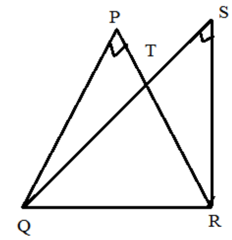
Hence the theorem is proved.

Converse of Pythagoras Theorem

Statement: If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is a right angle triangle.

5. Two triangle QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.

PTA-6



In ΔPQR and ΔSQR

$\angle P = \angle S = 90^{\circ}$ and ΔSQR

$\angle P = \angle S = 90^{\circ}$

And $\angle PTQ = \angle STR$ (vertically opposite angles)

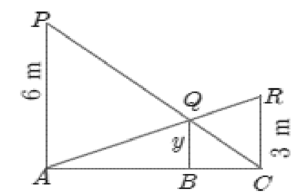
Thus by AA criterion of similarity we have $\Delta PTQ \sim \Delta STR$

$$\frac{PT}{ST} = \frac{TQ}{TR}$$

$\Rightarrow PT \times TR = TQ \times ST$

6. Two vertical poles of heights $6m$ and $3m$ are erected above a horizontal ground AC. Find the value of y.

PTA-5



In $\Delta PAC, \Delta QBC$ are similar triangles

$$\frac{PA}{QB} = \frac{AC}{BC} = \frac{PQ}{QC}$$

$$\frac{6}{y} = \frac{AC}{BC}$$

$y(AC) = 6BC$ ………………. (1)

ΔACR and ΔABQ are similar triangles

$$\frac{CR}{QB} = \frac{AC}{AB}$$

$$\frac{3}{y} = \frac{AC}{AB}$$

$3(AB) = (AC)y$……………. (2)

(1) & (2) $\Rightarrow 3AB = 6BC$

$$\frac{AB}{BC} = \frac{6}{3} = 2$$

$AB = 2BC$

$AC = AB + BC$

$AC = 2BC + BC$ $\quad (AB = 2BC)$

$AC = 3BC$

Substitute $AC = 3BC$ in (1) we get

$(3BC)y = 6BC$

$$y = \frac{6BC}{3BC}$$

$\boldsymbol{y = 2\ m}$

7. In figure $\angle QPR = 90^{\circ}$, PS is its bisector. If $ST \perp PR$, prove that $ST \times (PQ + PR) = PQ \times PR$.

PTA-2

Given: In the figure $\angle QPR = 90^{\circ}$,

PS is its bisector and $ST \perp PR$

$\frac{PQ}{PR} = \frac{QS}{SR}$ By Angle bisector theorem

$\frac{PQ}{PR} + 1 = \frac{QS}{SR} + 1$ Add 1 both side

$$\frac{PQ+PR}{PR} = \frac{QS+SR}{SR}$$

$\frac{PQ+PR}{PR} = \frac{QR}{SR}$………….. (1)

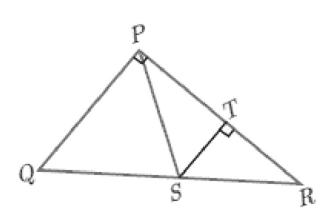
In ΔPQR and ΔSTR

$\angle QPR = 90^{0}$, $\angle STR = 90^{\circ}$

$\angle PRS = \angle TRS = \angle R$ is common ,

By AA similarity

$\therefore \frac{PQ}{ST} = \frac{QR}{SR} = \frac{PR}{TR}$ ………… (2)



(1) & (2) $\Rightarrow \frac{PQ+PR}{PR} = \frac{PQ}{ST}$

$ST\ (PQ + PR) = PQ \times PR$. Hence proved

8. **The hypotenuse of a right triangle is 6m more than twice of the shortest side. If the third side is 2m less than the hypotenuse, find the sides of the triangle.** PTA-3

In $\Delta ABC; \angle B = 90^0$

Let $AB = x \Rightarrow AC = 2x + 6$ *and*

$BC = 2x + 4$

$(2x+6)^2 = x^2 + (2x+4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16x + 16$

$x^2 + 16x - 24x + 16 - 36 = 0$

$x^2 - 8x - 20 = 0$

$(x-10)(x+2) = 0$

$x = 10$ (or) $x = -2$

But $x \neq -2$

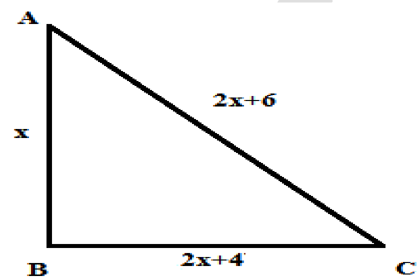
If $x = 10$

$\Rightarrow AC = 2x + 6$

$= 20 + 6 = 26$

$\Rightarrow BC = 2x + 4$

$= 20 + 4 = 24$



∴ The sides are $\boldsymbol{AB = 10m}$; $\boldsymbol{BC = 24m}$; $\boldsymbol{AC = 26m}$.

9. **The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that $QS = 3SR$. Prove that $2PQ^2 = 2PR^2 + QR^2$**

Given the ΔPQR, the perpendicular on the base QR at S, such that $QS = 3SR$

In $\Delta PQS \Rightarrow PQ^2 = PS^2 + QS^2$

$\Delta PSR \Rightarrow PR^2 = PS^2 + SR^2$

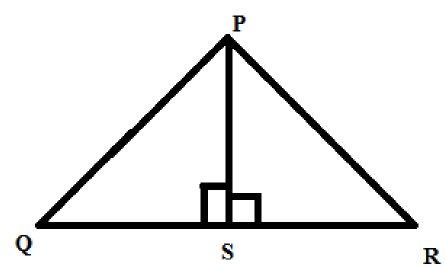
$\Rightarrow PS^2 = PR^2 - SR^2$

$QR = QS + SR$

$= 3SR + SR$

$QR = 4SR$

$\frac{QR}{4} = SR$



$PQ^2 = PR^2 - SR^2 + (3SR)^2$

$PQ^2 = PR^2 - SR^2 + 9SR^2$

$PQ^2 = PR^2 + 8SR^2$

$PQ^2 = PR^2 + \frac{8QR^2}{16}$

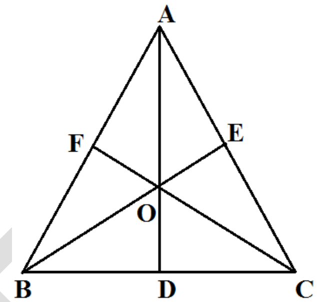
$\Rightarrow \boldsymbol{2PQ^2 = 2PR^2 + QR^2}$

10. Show that the angle bisectors of a triangle are concurrent.

PTA-4

In the ΔABC, "O" is any point inside the Δ

The angle bisector $\angle AOB, \angle BOC,$ and $\angle AOC$ meet the sides AB, BC & CA at D, E & F respectively.



$\therefore$ In $\Delta BOC, OD$ is the bisector of $\angle BOC$

$\therefore \frac{OB}{OC} = \frac{BD}{DC}$ ……………… (1)

Similarly in the triangle AOC & AOB we get

$\frac{OC}{OA} = \frac{CE}{AE}$ …………………….. (2)

$\frac{OA}{OB} = \frac{AF}{FB}$ ……………….. (3)

$(1) \times (2) \times (3) \Rightarrow \frac{OB}{OC} \times \frac{OC}{OA} \times \frac{OA}{OB} = \frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{EB}$

$\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1$ ……………… (4)

If AD, BE & CF are the bisectors of $\angle A, \angle B$ & $\angle C$ then by ABT

$\frac{AB}{AC} = \frac{BD}{DC}; \frac{BC}{CA} = \frac{AF}{FB}; \frac{AB}{BC} = \frac{AE}{EC}$

$\frac{AB}{AC} \times \frac{BC}{CA} \times \frac{AB}{BC} = \frac{BD}{DC} \times \frac{AF}{FB} \times \frac{AE}{EC}$

$1 = 1$ (By (4))

$\therefore O$ is the point of concurrent.

The angle bisectors of a triangle concurrent